University of Petra		
Faculty of Information Technology	جامعة البترا	كلية تكنولوجيا المعلومات
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Advanced Algorithms
601326
Midterm Exam – 2025 1

Your ...

Your Instructor Name: ……………………

Instructions for the Exam:

- Write your name and ID number on the exam and answer sheets.
- Write the number of the section that you enrolled in.
- Write the name of your instructor.
- Questions in the exam not allowed.
- Using any type of technology (mobiles, smart watches) not allowed
- Using extra papers or sheets not allowed
- The exam consists of Six questions.

For instructor use only:

Question number	Course ILO	Program ILO	Question weight	Student mark
Q1			4	0.75
Q2	K2		4	3.5
Q3	I1		8	0.5
Q4			5	2.75
Q5			5	4.5
Q6			4	3.5
Total /30				15.5

Q1) <u>Discuss median-of-three pivot selection and how it enhances</u> Quick Sort algorithm
worst case. **(4 marks)** (0.75)

it will take the pivot from the

middle $O(n^2)$!

it will divide the array multiple times
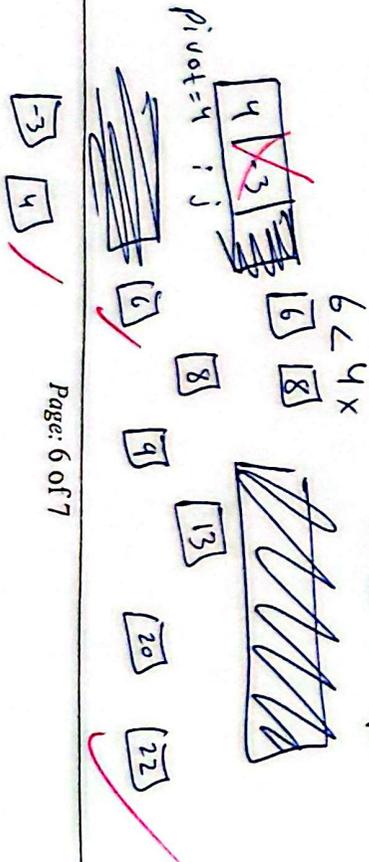
which might make it easier

How?!

Q2) Consider the following algorithm.   **(4 marks)**

Algorithm XYZ (A [0..n – 1])

//Input: An array A[0..n – 1] of n real numbers

```
val ← 10      → |
res1 ← 0      → |
res2 ← 1      → |
for i ← 0 to n – 1 do    → n+3
    if A[i] >= val  → | (n-1)
        res1 ← A[j]+res1  → | (n-1)
    if A[i] < val/2  → | (n-1)
        res2 ← A[j]*res2  → | (n-1)
return res1 – res2    → |
```

a. What does this algorithm compute?
b. What is the time complexity for the algorithm?

$$T(n) = 1 + 1 + 1 + n + 3 + n - 1 + n - 1 + n - 1 + n - 1 + n - 1 + 1$$

$$3 + 5n$$

$O(n)$

it will add the numbers in the array if $A[i] >= value$
give u the sum and store it in res1

and will multiply the numbers in the
array if $A[i] < val/2$, and store the result in
res 2

arr

8	5	3	10	7	2	1

value = 10

res1 = 6

res 2 = 1

if 8 >= 10
  8 + 0
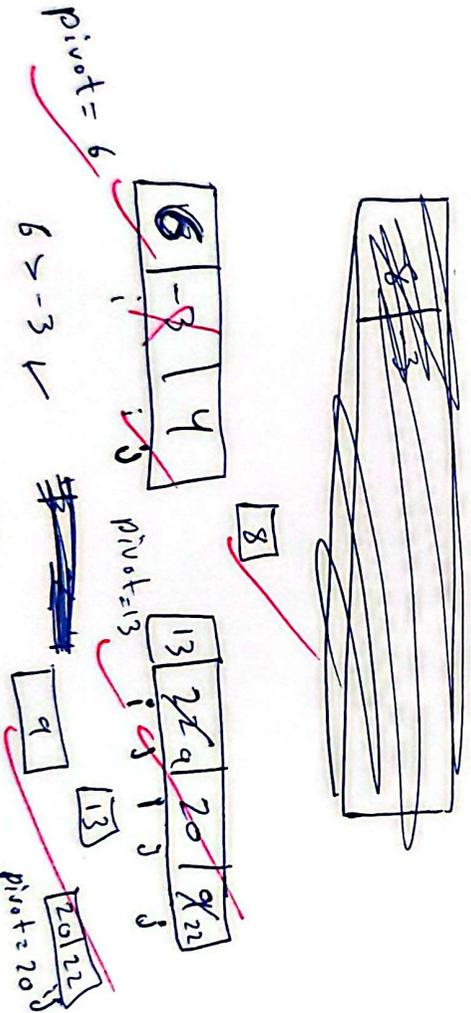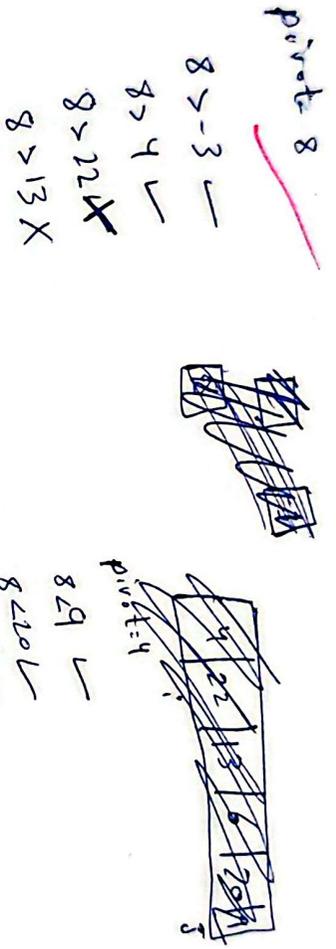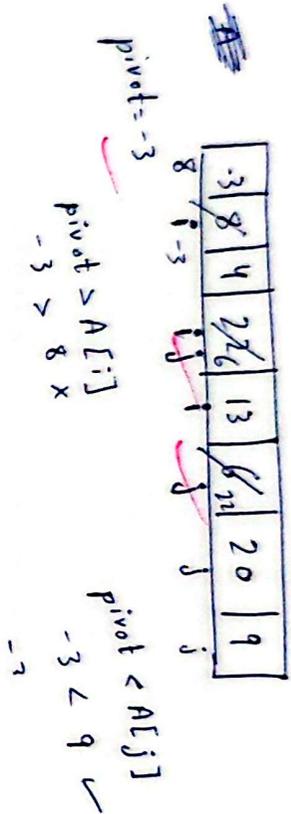if 8 < 10/2
  8 * 1 = 8

3.5

1.5

Q3) Design a divide-and-conquer algorithm to compute the summation of elements in an array. **(8 marks)**

Setup and solve a recurrence relation for the number of summations made by your algorithm (use Master Theorem)

Divid & conquer (A, n)

res = Arr[] ⟶ 1

For i=0 to n-1 ⟶ $n+3$

res += A[i] ⟶ $1(n-1)$

return res ⟶ 1

$T(n) = 1 + n + 3 + n-1 + 1$

$O(n)$

Q4) Consider the following recursive algorithm:

(5 marks)

```
Algorithm Q(n)
//Input: A positive integer n
if n =1
    return 1
else
    return Q(n − 1) + 2 * n − 1
```

a) **Setup a recurrence relation for the number of multiplications made by this algorithm.**

b) **Solve the recurrence relation using backward substitution and find Big O.**

$T(n) = \begin{cases} 1 & , n=1 \\ T(n-1) \times 2 & ? \end{cases}$ **0.75**

$T(n) =$

$T(n) = T(n-1) + 2 \times n - 1$
$T(n-1) = T(n-2) + 2 \times n - 2$
$T(n-2) = T(n-3) + 2 \times n - 3$
$T(n-3) = T(n-4) + 2 \times n - 4$

$T(n) = T(n-4) + 2(n-4) + 2(n-3) + 2(n-2) + 2(n-1)$
$T(n-4) + 2n - 8 + 2n - 6 + 2n - 4 + 2n - 2$
$T(n-4) + 8n - 20$

let $i = 4$

$T(n-i) + 2^{i} - 8(n) - i^2 + 4$

$T(n-i) = T(1)$ $n - i = 1$
$+i 1$

$1 + 2^{n} - 8n - n^2 + 4$
$5 + 2^{n} - 8n - n^2 + 4$
$O(n^2)$

$T(i-1) + 2^{n} - 8n - n^2 + 4$

②

2.75

Q5) Given the following array, A= -3, 8, 4, 22, 13, 6, 20, 9
Apply Quick Sort algorithm to sort the array elements in ascending order (show detailed steps)

(5 marks)

-3	8	4	22	13	6	20	9

$$\begin{array}{cccccccc} -3 & 8 & 4 & 22 & 13 & 6 & 20 & 9 \\ i & & & & & & & j \end{array}$$

pivot = -3

pivot > A[i]
-3 > 8 X

pivot < A[j]
-3 < 9 ✓
-3

pivot = 8

8 > -3 ✓
8 > 4 ✓
8 > 22 ✗
8 > 13 X

$$\begin{array}{cccc} 4 & 22 & 13 & 6 & 20 & 9 \\ i & & & & & j \end{array}$$
pivot = 4

8 < 9 ✓
8 < 20 ✓
8 < 6 X
8 < 13 ✓

pivot = 8

-3	8	4
	i	j

$$\begin{array}{ccc} -3 & 8 & 4 \\ & i & j \end{array}$$

pivot = 6

6 > -3 ✓
6 > 3 ✓

6 < 4 ✗
6 < 4 X
6 < 9 ✓

13	22	9	20	9

pivot = 13
pivot = 20

9		13

| 20 | 22 |

pivot = 20

pivot = 4
4 3

-3	4

pivot = 4   i = j

6	8	9	13	20	22

-3		4

Q6) Based on your understanding of the covered algorithms, answer the following:
(4 marks)

3.5

a) Compare Selection Sort to Sequential Search algorithms in terms of best and worst cases.

	Selection sort	Sequential search
Best case	n ✗	1 ✓
Worst case	n^2 ✓	n ✓

1.5

b) Discuss briefly the Best Case of Insertion Sort algorithm.

if the array was already sorted, $O(n)$

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